



POWER OPTIMIZATION OF AN EXTRA-TERRESTRIAL, SOLAR-RADIANT STIRLING HEAT ENGINE

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Abstract—The power output and thermal efficiency of a finite-time, optimized, extra-terrestrial, solar-radiant Stirling heat engine have been studied. The thermodynamic model adopted is a regenerative gas Stirling cycle coupled to a heat source and heat sink by radiant heat transfer. Both the heat source and sink are assumed to have infinite heat-capacity rates. Expressions are obtained for optimum power and efficiency at optimum power for a cycle based on higher and lower temperature bounds.

INTRODUCTION

Since its conception in the early 1800s, the Stirling engine has periodically enchanted engineers and physicists because of the theoretical potential for approaching the Carnot efficiency.^{1,2} Today, interest in the Stirling engine is again on the rise.³ Among the reasons for this are great advances in materials technology, inherent environmental advantages of the Stirling engine and the fact that, as an externally-heated engine, it can be powered by a number of means.⁴ The Stirling engine is either currently being used or is under consideration for use in a wide variety of applications,⁵⁻⁸ including both extra-terrestrial and terrestrial solar installations.^{9,10}

For lack of an alternative procedure, practical engineers have typically made use of the thermodynamic air-standard approach to model and study trends in applications of real gas Stirling engines. This analysis provides exceedingly generous estimates of the potential performance of such engines and is not relevant in the design activity. Thus, to provide a more reasonable estimate of the performance potential of a real regenerative solar-radiant Stirling engine, the innovative finite-time thermodynamics¹¹ approach is employed herein on an endoreversible version of the Stirling cycle. An endoreversible cycle is one in which the external heat-transfer processes are considered the only irreversible processes within the cycle.¹² The preferred use of finite-time thermodynamics in this study is based on the inherent reality that a heat engine must produce work in a limited amount of time. In order to accomplish this task, the external heat-transfer processes of the cycle are modeled as occurring across finite-temperature differences and are thereby considered irreversible. The endoreversible cycle is then optimized with respect to power rather than efficiency (the usual focus of thermodynamic analysis). For clarity, this study is thus based on the recognition that real engines do not produce work over an infinite time but must produce power. Work produced over an infinite amount of time gives zero power.

The relationships developed and the analysis obtained should provide insight into the operational and design criterion necessary for the attainment of optimum power in a simple solar-radiant Stirling engine for use in space applications. For this baseline study, only black-body surfaces are considered in the external heat-exchange process by radiation. The discussion is limited to a thermodynamic analysis of the engine itself and thus does not include a detailed description of the actual hardware involved in this energy-conversion process.

THE ENDOREVERSIBLE EXTRA-TERRESTRIAL, SOLAR, RADIANT STIRLING ENGINE AND ITS FINITE-TIME POWER OPTIMIZATION

The regenerative endoreversible Stirling cycle is depicted in Figs. 1 and 2. This cycle approximates the compression stroke of the real engine as an isothermal process 1-2, with irreversible heat rejection

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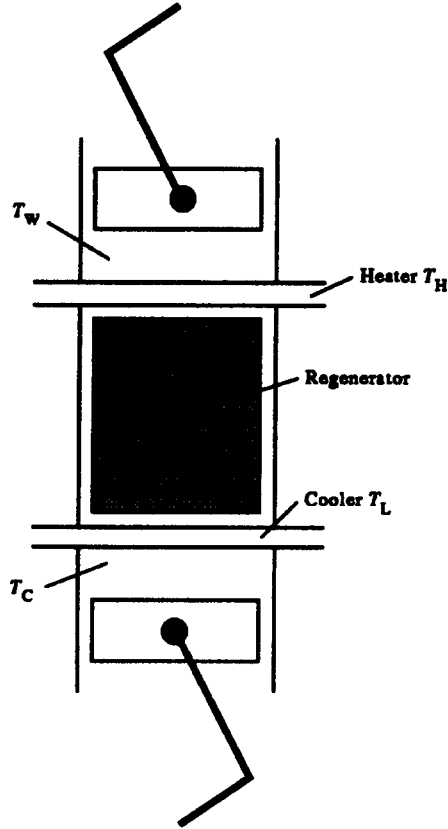


Fig. 1. Functional schematic of the regenerative Stirling engine; note that the opposing pistons are 90 degrees out of phase.

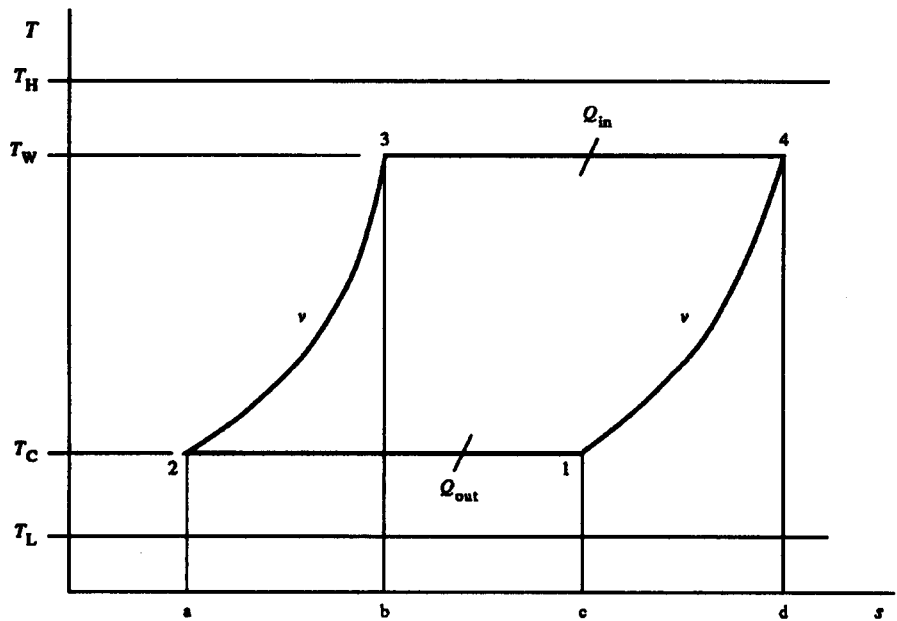


Fig. 2. Temperature-entropy diagram of the regenerative Stirling engine (ideal).

to an intermediate low-temperature sink. The heat addition to the working fluid from the regenerator is modeled as a reversible constant-volume process 2–3. The work-producing expansion stroke is modeled as an isothermal process 3–4, with irreversible heat addition from an intermediate high-temperature source. Finally, heat rejection to the regenerator is modeled using a reversible constant-volume process 4–1.

Within real Stirling engines, the external heat-transfer processes 1–2 and 3–4 must each occur in finite times. This in turn requires that these heat-transfer processes must proceed through finite-temperature differences and are therefore defined as being externally irreversible. During the first of these, the heat rejection process 1–2, energy flows from the working fluid which is maintained at a constant temperature T_c , to the intermediate low temperature sink T_L , thus flowing across the temperature difference $(T_c - T_L)$, as shown in Fig. 1. Similarly, during the heat addition process 3–4, heat is transferred from the intermediate high temperature sources T_H to the working fluid which is maintained at a constant temperature T_w , thus flowing across the temperature difference $(T_H - T_w)$.

For satellites in our solar system, the ultimate heat source is the sun (at temperature T_s) and the ultimate heat sink is outer space (at temperature T_o). If T_H is the temperature at the external (radiative) surface of the black-body heater plate, then heat is transferred to this surface across the temperature difference $(T_s - T_H)$ and from this surface across $(T_H - T_o)$. Similarly, if T_L is the temperature of the external surface of the cooler plate, heat transfer to the universe from this surface takes place across the temperature difference $(T_L - T_o)$.

If the regeneration is assumed to be ideal, the heat rejected to the regenerator by the hot working fluid during the process 4–1 is equivalent to the heat supplied to the cold working fluid during 2–3. This can be visualized by noting that the area under the process path on the temperature–entropy diagram of Fig. 2 for process 4–1, area c–1–4–d–c, is equal in magnitude to the area under process path 2–3, area a–2–3–b–a. The reasonableness of this validated by the fact that assumption is the efficiency of regenerators is continuing to improve, with at least one manufacturer as far back as 1972 reporting regenerator effectiveness values of 95%,¹³ and progressing to reported values of 98–99%.¹⁴ Thus, the relation for the overall ideal regenerative heat-transfer processes is

$${}_2Q_3 = {}_4Q_1 \quad (1)$$

Since the regenerator supplies the thermal energy transferred to the working fluid during 2–3, the only source of external heat addition to the cycle is provided by the intermediate source during process 3–4. If the working fluid is considered to be an ideal gas, the magnitude of heat addition during 3–4 can be found by substituting the definition for boundary work into the conservation of energy relation, yielding

$$Q_{in} = {}_3Q_4 = mRT_w(\ln r_v) \quad (2)$$

where r_v = compression ratio = V_1/V_2 . Similarly, the external heat rejection 1–2 is

$$Q_{out} = -{}_1Q_2 = mRT_c(\ln r_v) \quad (3)$$

For the thermodynamic cycle 1–2–3–4–1, conservation of energy yields

$$W_{net} = Q_{net} = Q_{in} - Q_{out} \quad (4)$$

Substituting Eqs. (2) and (3) into Eq. (4) gives

$$W_{net} = mR(\ln r_v)(T_w - T_c) \quad (5)$$

The average net power for a cycle is found by dividing the net cycle work by the time for one cycle t . This approach yields the following equation for the net cycle power:

$$P = W_{net}/t = mR(\ln r_v)(T_w - T_c)/t \quad (6)$$

The total cycle time is the sum of the individual process times, i.e.

$$t = t_{12} + t_{23} + t_{34} + t_{41} . \quad (7)$$

An order of magnitude analysis⁴ shows that the times required to accomplish the isothermal compression and expansion processes are of the same order of magnitude as the times required to accomplish the constant volume processes; therefore all four times in Eq. (7) must be evaluated.

Using Eqs. (2) and (3), the average rates of external heat transfer into and out of the engine are given by

$$Q_{in} = Q_{in}/t_{34} = mRT_w(\ln r_v)/t_{34} \quad (8)$$

and

$$Q_{out} = Q_{out}/t_{12} = mRT_c(\ln r_v)/t_{12} . \quad (9)$$

The rates of external heat transfer into and out of the engine may further be quantified from heat transfer theory as being proportional to the temperature between the respective intermediate thermal reservoirs and the constant temperature states of the working fluid during each process. One more set of expressions for these rates can be found from quantifying the black body exchange between these intermediate reservoirs and their respective ultimate thermal reservoirs. All of this implies that Eqs. (8) and (9) can also be expressed as

$$Q_{in} = U_w A_w (T_H - T_w) = A_H \sigma [F_{HS} T_S^4 + (1 - F_{HS}) T_O^4 - T_H^4] = A_H \sigma (F_{HS} T_S^4 - T_H^4) \quad (10)$$

and

$$Q_{out} = U_c A_c (T_c - T_L) = A_L F_{LO} \sigma (T_L^4 - T_O^4) , \quad (11)$$

where, on the thermal side internal to the engine assembly, U represents an average overall heat-transfer coefficient due to conduction and convection between a radiative surface and the cycle working substance and where the effective areas, A_w and A_c , represent heat transfer areas appropriate to the proper specification of thermal energy exchange between these radiative surfaces and the cycle working substance during the respective warm and cold constant temperature processes. From the radiation side, A_H and A_L are the system areas receiving and emitting net radiant thermal energy, respectively. Also, σ is the Stefan-Boltzmann constant and F_{HS} and F_{LO} are the view factors defined as the fractions of the energy leaving the surfaces represented by the first subscripts and reaching the thermal reservoirs represented by the second subscripts.

Substituting Eqs. (10) and (11) into Eqs. (8) and (9), respectively, yields expressions for the times associated with the external heat-transfer processes, viz.,

$$t_{12} = mRT_c(\ln r_v)/[U_c A_c (T_c - T)] = mRT_c(\ln r_v)/[A_L F_{LO} \sigma (T_L^4 - T_O^4)] , \quad (12)$$

and

$$t_{34} = mRT_w(\ln r_v)/[U_w A_w (T_H - T_w)] = mRT_w(\ln r_v)/[A_H \sigma (F_{HS} T_S^4 - T_H^4)] . \quad (13)$$

Equation (6) yields expressions for the power which are based upon reservoir and isothermal working fluid temperatures.

From Ref. 4, the times associated with the regenerative heat-transfer processes can be expressed as

$$t_{23} = t_{41} = mc_v (T_w - T_c)/[U_{reg} A_{reg} (1 \text{ K})] . \quad (14)$$

Equations (12)–(14) may then be substituted in turn into Eq. (6), yielding expressions for the power which are based upon reservoir and isothermal working fluid temperatures, viz.,

$$P = \{T_w/[U_w A_w (T_H - T_w)(T_w - T_c)] + T_c/[U_c A_c (T_c - T_L)(T_w - T_c)] + 2c_v/[R(\ln r_v)U_{reg}A_{reg}]\}^{-1} \quad (15a)$$

and

$$P = \{T_w/[A_H \sigma (F_{HS} T_S^4 - T_H^4)(T_w - T_c)] + T_c/[A_L F_{LO} \sigma (T_L^4 - T_O^4)(T_w - T_c)] + 2c_v/[R(\ln r_v)U_{reg}A_{reg}]\}^{-1} \quad (15b)$$

We note from Eq. (15) that power is a function of the variable temperatures T_w , T_H , T_c , and T_L . The maximum power with respect to these four as yet undetermined temperatures can be obtained through the simultaneous solution of the following set of equations:

$$\partial p / \partial T_w = 0, \quad (16a)$$

$$\partial p / \partial T_c = 0, \quad (16b)$$

$$\partial p / \partial T_H = 0, \quad (16c)$$

and

$$\partial p / \partial T_L = 0, \quad (16d)$$

Application of Eqs. (16a) and (16b) to Eq. (15a) results in functional relationships between the working substance temperatures and the intermediate heat reservoir temperatures necessary for maximum power, namely,

$$(T_c)_{opt} = CT_L^{0.5} \quad (17)$$

and

$$(T_w)_{opt} = CT_H^{0.5}, \quad (18)$$

where

$$C = [(U_w A_w T_H)^{0.5} + (U_c A_c T_L)^{0.5}] / [(U_w A_w)^{0.5} + (U_c A_c)^{0.5}]. \quad (19)$$

Proper substitution of these three results into Eq. (15b) enables the formulation for power to be recast into two new expressions, one in terms of the independent variables T_H and T_L only and the other in terms of the independent variables T_c and T_w only. The maximum power can then be obtained either by application of the Eqs. (16c) and (16d) to the first of these expressions or application of the Eqs. (16a) and (16b) to the second. To ensure the integrity of the work, both routes were employed yielding the same results for $(T_L)_{opt}$ and $(T_H)_{opt}$.

Assuming T_O to be absolute zero,

$$(T_L)_{opt} = \{7A_H[F_{HS}T_S^4 - (T_H)_{opt}^4]/[A_L F_{LO}(F_{HS}T_S^4 + 7(T_H)_{opt}^4)]\}^{1/4} \quad (20)$$

and

$$(T_H)_{opt} = \{A_H[F_{HS}T_S^4 - (T_H)_{opt}^4][F_{HS}T_S^4 + 7(T_H)_{opt}^4]/(7^7 A_L F_{LO})\}^{1/36}. \quad (21)$$

Equation (21) is transcendental in nature and can be solved using a simple iterative numerical technique. The values given by Eqs. (17), (18), (20), and (21) had to be tested using a combination of techniques to ensure that they do in fact yield maximum (instead of minimum) values. Based on Eq. (15a) and these last two results, the expression for optimum power can be expressed as:

$$P_{\text{opt}} = \{ [(U_c A_c)^{0.5} + (U_w A_w)^{0.5}]^2 / [U_c A_c U_w A_w (T_H)_{\text{opt}}^{0.5} - (T_L)_{\text{opt}}^{0.5}]^2 + 2c_v / [U_{\text{reg}} A_{\text{reg}} R (\ln r_v)] \}^{-1} \quad (22)$$

The work per cycle at maximum power or work optimum is determined by substituting the optimum working fluid temperature values from Eqs. (17) and (18) into Eq. (5), resulting in

$$W_{\text{opt}} = C m R (\ln r_v) [(T_H)_{\text{opt}}^{0.5} - (T_L)_{\text{opt}}^{0.5}], \quad (23)$$

where C is defined in Eq. (19).

Finally, the thermal efficiency of the cycle at the condition of optimum power is found by first substituting the optimum warm fluid temperature, Eq. (18), into Eq. (2) for the external heat transfer to the working fluid. This result and the optimum work given by Eq. (23) are then substituted into the basic definition for thermal efficiency, yielding

$$\begin{aligned} \eta_{\text{opt}} &= W_{\text{opt}} / (Q_{\text{in}})_{\text{opt}} = R (\ln r_v) [(T_H)_{\text{opt}}^{0.5} - (T_L)_{\text{opt}}^{0.5}] / [(T_H)_{\text{opt}}^{0.5} R \ln(r_v)] \\ &= 1 - [(T_L)_{\text{opt}} / (T_H)_{\text{opt}}]^{0.5}, \end{aligned} \quad (24)$$

$$\eta_{\text{opt}} = 1 - (T_c / T_w)_{\text{opt}} = 1 - [(T_L / T_H)_{\text{opt}}]^{0.5} < 1 - T_O / T_H = \eta_{\text{th}}, \quad (25)$$

where $1 - (T_O / T_H)$ is the efficiency that could be obtained if the process were allowed to take place over infinite time in a cycle using an ideal regenerator and where $(T_H) = T_S (\sigma F_{\text{HS}})^{0.25}$. Thus the efficiency at optimum power will always be a value that is substantially less than this upper bound.

NUMERICAL EXAMPLE FOR A SOLAR SATELLITE

Consider a satellite solar-radiant Stirling engine which receives radiant heat from the sun ($T_S = 5755$ K) and emits radiant heat to space ($T_O = 0$ K). The radius of the sun (R) is 6.95×10^8 m and the distance between the sun and the satellite (L) is 1.49×10^{11} m. $F_{\text{HS}} = R^2 / (R^2 + L^2) = 2.176 \times 10^{-5}$ and $F_{\text{LO}} = 1$. Thus the expression $F_{\text{HS}} \sigma T_S^4$ contained in Eq. (10) has a power density of 1.353 kWm^{-2} which is known to be the solar constant. Values for A_L and A_H can be found through the use of Eqs. (12) and (13), respectively, once the other values contained in these expressions are specified. Note however, that these equations contain unknown temperature values. Yet, for initial computations A_L and A_H are not needed if time symmetry between the external heating process, 3–4, and the cooling processes, 1–2, is maintained. This symmetry condition requires that

$$A_H / A_L = F_{\text{LO}} (T_L^4 - T_O^4) / [(F_{\text{HS}} T_S^4 - T_H^4) T_L^{0.5}], \quad (26)$$

which can be used in turn to eliminate A_H and A_L from Eqs. (20) and (21). As noted in Ref. 14, time symmetry between these two processes also requires that $U_w A_w = U_c A_c$. To be comparable with typical existing Stirling engines,⁴ values of $U_w A_w$ in the range 1–4 kW/K were considered for an engine with a regenerator $U_{\text{reg}} A_{\text{reg}}$ value of 1000 kW/K, with a compression ratio (r_v) of 2 and with 9.3×10^{-3} kg of air as the working fluid. A c_v value of 0.7165 kJ/kg(K) and an R value of 0.287 kJ/kg(K) were used in the calculations.

For the above installation, the values of $(T_H)_{\text{opt}}$, $(T_w)_{\text{opt}}$, $(T_c)_{\text{opt}}$, and $(T_L)_{\text{opt}}$ were found to be the constant values 330.5, 293.8, 228.5 and 200.0 K, irrespective of $U_w A_w$. Figure 3 contains a plot of P_{opt} vs $U_w A_w$ for this engine. The thermal efficiency of this power optimized engine is 0.222 for all $U_w A_w$ values, compared to a theoretical infinite-time upper-bound value of 1.0 for a Stirling engine operating between 5755 and 0 K and also using an ideal regenerator.

A parametric study was conducted using this formulation to analyze the effect of F_{HS} on η_{opt} and P_{opt} . It was found even though increasing F_{HS} by some means (such as by use of a parabolic reflector) can improve the power (Fig. 3), the value of η_{opt} remains unchanged. For a F_{HS} of 8.0, for example, values for $(T_H)_{\text{opt}}$, $(T_w)_{\text{opt}}$, $(T_c)_{\text{opt}}$ and $(T_L)_{\text{opt}}$ were found to be 555.9, 494.1, 384.3 and 336.3 K, respectively and the values for the power for different values of $U_w A_w$ were found to be approximately 1.682 times those found for corresponding $U_w A_w$ values the unenhanced F_{HS} case (Fig. 3). However η_{opt} is

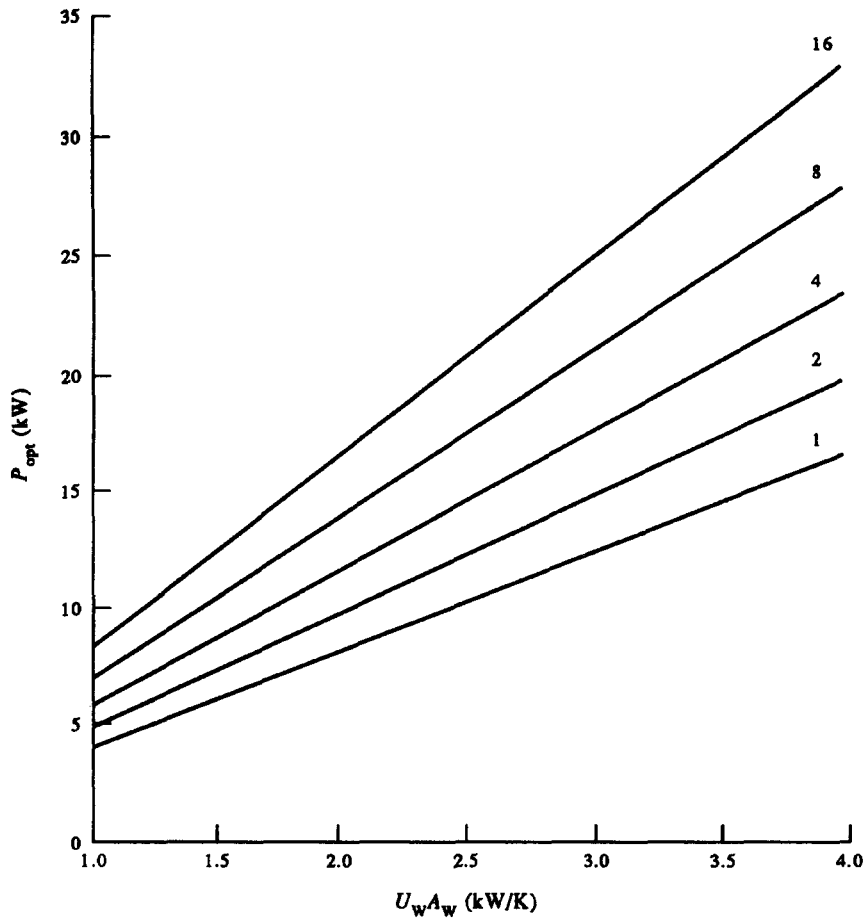


Fig. 3. Optimum power vs heat conductance ($U_w A_w$) for various shape factors (F_{HS}).

0.222 for this F_{HS} for all $U_w A_w$ values also. Thus one can see that an η_{opt} of 0.222 represents the upper limit for an engine with the above internal characteristics, performing at optimum power and using a simple black-body collector.

CONCLUSIONS

This work demonstrates the utility of a finite-time analysis approach in both the study and optimization of an extra-terrestrial, solar-radiant Stirling heat engine. The efficiency at optimum power obtained for the model studied is much more realistic than that provided by traditional thermodynamic techniques involving infinitely large cycle times. Also, while the latter approach results in the production of zero power, the finite-time approach, given herein, predicts very reasonable values.

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NOMENCLATURE

A_c, A_w = Effective heat-transfer surface areas between the endoreversible Stirling engine and the intermediate low- and high-temperature reservoirs, respectively	T_c = Temperature of the working fluid during the isothermal external heat-rejection process to the intermediate low-temperature sink
A_H, A_L = Heat-transfer surface area of the heater and cooler plates, respectively	T_H, T_L = Temperatures of the intermediate heat source and intermediate heat sink, respectively
A_{reg} = Heat-transfer surface area of the Stirling engine regenerator	T_O = Temperature of outer space (the low-temperature heat sink)
C = Heat-engine parameter defined by Eq. (19)	T_s = Temperature of the sun (high-temperature heat source)
c_v = Specific heat at constant volume (kJ/kg-K)	T_w = Temperature of the working fluid during the isothermal external heat-input process from the intermediate high-temperature source
F_{HS}, F_{LO} = Shape factors between the subscripted heat-transfer surfaces (A_H and A_L) and the subscripted thermal reservoirs (S—sun, O = outer space)	t = Total cycle time
L = Distance between the Stirling engine and the high-temperature reservoir	t_{12} = Time required to accomplish the isothermal compression
m = Mass of working fluid in the engine	t_{23} = Time required for the heat-transfer process from the regenerator to the working fluid
P, P_{opt} = Net cycle power output and optimum net cycle power output, respectively	t_{34} = Time required to accomplish the isothermal power stroke
$Q_{in,3}, Q_4$ = Isothermal heat transfer into the engine from the intermediate high temperature source	t_{41} = Time required for the heat-transfer process from the working fluid to the regenerator
$Q_{out,1}, Q_2$ = Isothermal heat transfer from or to the engine to or from the intermediate low temperature sink, respectively	U_c, U_w = Overall heat-transfer coefficient between the engine and the intermediate low-temperature sink or the intermediate high-temperature source, respectively
${}_2Q_3$ = Heat transfer from the regenerator to the working fluid	U_{reg} = Overall heat-transfer coefficient to and from the regenerator and the working substance during the constant-volume processes
${}_4Q_1$ = Heat transfer captured by the regenerator from the working fluid	v = Specific volume
Q_{in} = Rate of heat transfer into the engine from the high-temperature source	V_1, V_2 = Volume at bottom dead center and top dead center, respectively
Q_{out} = Rate of heat transfer into the engine from the low-temperature sink	W_{net}, W_{opt} = Net work output for one cycle and net work output per cycle at the optimum power condition, respectively
R = Gas constant; radius of the sun	η_{opt}, η_{th} = cycle thermal efficiency at the optimum power condition and infinite time, respectively
r = Compression ratio, V_1/V_2	σ = Stefan-Boltzmann constant
s = Specific entropy	